# Examples 9 in Ratios and Rates



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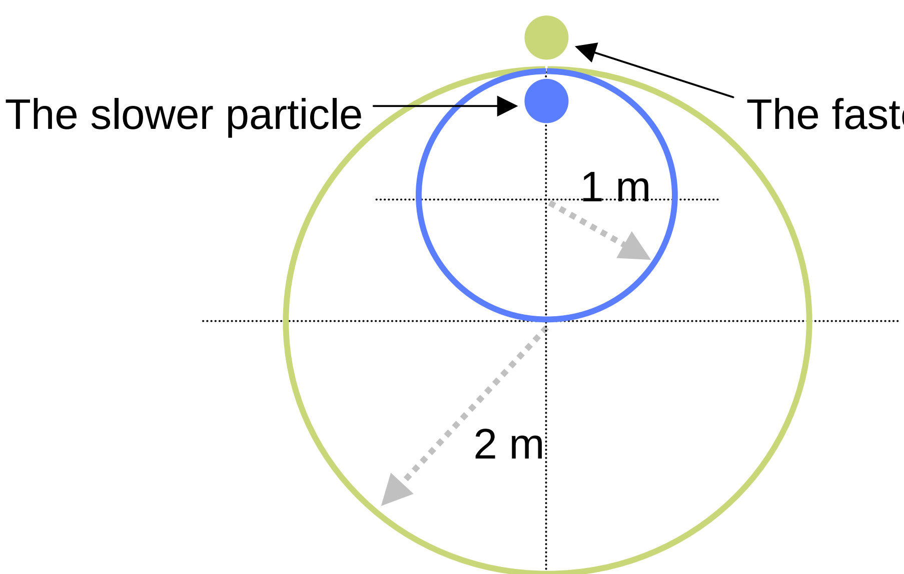
As in Fig. 0 below, one particle is orbiting at 10 m/s along a circular path of the radius 2 m, and the other particle is at 8 m/s along the other path with 1 m in radius. Both move in the same direction. The smaller path is inscribing the larger, so both paths meet at one point. Suppose now both just have started moving at the point where the paths meet. Then, see if the two particles will meet. If not, what can we do to make particles meet?

Fig. 0



We may want to begin with what this problem is made of. We can begin with examining the objects making up the problem. So to begin with, what are the objects?

They constitute the problem, and for instance, include the speeds of the particles and the radii of the orbits. Thus, make use of what’s given in the problem description. Making use of it, we can make questions as follows.

Where can the particles meet? In other words, where will their meeting have to happen?
When can they meet?
When do they begin orbiting?
Where do they begin moving?
In what direction do they orbit?
Then, the possible answers can be as follows.

The meeting will have to happen at the point where the orbits meet.
They will meet at the same time, of course.
They begin orbiting at the same time.

Both particles start at the position where the orbits meet
And each particle moves in one direction, but the direction doesn't matter in this case.
Suppose now the point where the orbits meet is the target point. Then, if the particles meet, they get to the target point at the same time. How then do they get there concurrently?

Let's now keep track of movements of the particles.
Suppose first, when one particle arrives at the target point, the other is not there. What then does the one have to do?

Of course, it has to keep moving until it meets the other.
How then do they move along their orbits?

The particles move at their speeds, but get to the target point at the same time if they meet, of course. How much time then do they spend until the meeting?

The same amount of time. At different speeds though.
What then is different?
The distances covered. How then do we get the distances?

We can express the distance each particle has to cover along its orbit until it meets the other.
What then does the distance covered have to do with?

The speed and the time taken.
For each particle thus, we have three objects to work on. One is the distance covered, another is the speed, and the other is the time taken to cover the distance until the meeting. We have another way though, to get the distance to be covered until the meeting in this case.  How?

The particle can cover the distance the way as follows, too.

It moves along the track, and gets to the target point. If the other particle is not there at the time of its arrival at the target point, it continues moving, and gets to the target point again. And it moves along the same path over and over until the meeting.  What then does the particle make?

Laps. And of course, the same is true to the other one, too.

So the particles keep making their laps while orbiting. Thus, we want to find how many laps they have to make until the rendezvous, and then, we can express the distance.
How then can we get the numbers of laps?

We may want to first set up objects representing the number of the laps to be made. So suppose now A is the faster particle, B is the slower particle, u is the number of laps A makes, and v is the number of laps B makes.
Then, we now want to know how many laps both will have to make until they meet. Then, we want to consider two facts as follows.
Fact 1.
Each particle makes a number of its laps until the meeting.
Fact 2.
They begin orbiting at the same time, and meet at the same time. Thus, both will spend the same amount of time until the meeting. And each moves at its speed.

So suppose now that the same amount time = t.
Then, what can we draw from the two facts above?

They need to make their particular numbers of the laps in the time t until the meeting. So overall, we have the setting as follows.
Until the meeting, A will make a particular number of its laps in t, and B will also, make a particular number of its laps in t.

So we can use the numbers of the laps, and use the amount of time t to be taken until the meeting, along with the speeds. How?

A will cover a particular distance in a particular number of the laps in time t. Also, A will cover the same distance orbiting at its speed in time t. And the same applies to B, too. So for each particle, we can describe the distance covered at each speed in t, and the same distance is covered by the number of the laps in t.

We know the distance = the speed x the amount of time.
So using the speeds and the time, we can get the distances.
Suppose now F is the distance covered in t by the particle A, and S is the distance covered in t by the particle B.

Then, we get $F = 10t$, and $S = 8t$.

Let's now move on to the use of the laps. What then does the number of the laps have to do with the distance?

We know that each particle has to make a particular number of laps ($u$ or $v$) until the meeting, and that each lap takes up the length of an orbit. And we have the information on the lengths of both orbits.
So we can describe each distance using the number of the laps and the length of each lap. Then, each distance is the number of the laps x the length of the lap.
We have $u$ = the number of the laps A makes, and $v$ = the number of the laps B makes until the meeting.

So $F = u$ x the length of the orbit of A,
and $S = v$ x the length of the orbit of B

And the length of each orbit is the circumference of a circle. So assuming C is the circumference of a circle, D is the diameter, and r is the radius, we get $C = \pi D = \pi \times 2 \times r = 2\pi r$.

Thus, the distance covered by each lap = $2\pi r$.

We know F is the distance covered in t by the particle A, the radius of the orbit is 2 meters, and the number of the laps made until the meeting is u.
So $F = u \times 2\pi r = u \times 2\pi \times 2 = 4\pi u$, which is the distance A covers after its u laps in time t.

We know S is the distance covered in t by the particle B, the radius of the orbit is 1 meter, and the number of the laps made until the meeting is v.
So $S = v \times 2\pi r = v \times 2\pi \times 1 = 2\pi v$, which is the distance B covers after its v laps in t.

Thus, we have $F = 4\pi u$ and $S = 2\pi v$.
And we have $F = 10t$, and $S = 8t$, too.

We know each particle makes u or v laps in t.
So the distance covered at 10 m/sec in t by A is the same as the distance covered by u laps made by A.

And the distance covered at 8 m/sec in t by B is the same as the distance covered by v laps made by B.

Thus, we have $10t = 4\pi u$ and $8t = 2\pi v$.
So we now have 2 equations, but the equations have 3 unknowns, which means many solutions. The solution is not unique. What then can we do?

We can take the ratio between u and v.
So taking the ratio, we can get first,
$10t = 4\pi u \Rightarrow u = 5t/2\pi$, and $8t = 2\pi v \Rightarrow v = 4t/\pi$.
Thus next, we get $u:v = (5t/2\pi):(4t/\pi) = (5/2):4 = 5:8$, since $t/\pi$ is a common divisor. So we get $u:v = 5:8$.

So in every time interval t in which the faster particle makes 5 laps, and the slower particle makes 8 laps, they get to meet at the target point where the two orbits meet.

Let's now find the amount of time interval t, too.
Setting $u = 5$, we get $10t = 4\pi u = 20\pi \Rightarrow t = 2\pi$ sec.
Setting $v = 8$, we get $8t = 2\pi v = 16\pi \Rightarrow t = 2\pi$ sec, too.

So at every $2\pi$ (approximately 6.28) seconds, the particles meet at the point where the two orbits meet.
Therefore, the two particles meet every time the faster particle makes 5 laps, and the other makes 8 laps in every $2\pi$ seconds.
Now, we may want to ask ourselves if the particles will meet even if they move at any constant speed or on paths with any lengths or shapes.

Of course, when we take an exam and fill up the answer sheet, we don't need to put all the procedures stated above.

We need to show the basis on which our calculations have been performed to produce any particular numbers. In fact, we should be able to provide a summary on the procedures that we have taken to solve the problem.

The summary needs to have essential facts that can explain the outcomes of the significant subsequent procedures.
The answer sheet should look like as follows, for instance.

Suppose the particles will meet, one is A, and the other is B. They will meet at the same time at the point where the paths meet.

Then, they will have to make laps around their orbits, and have to travel the same amount of time.

Suppose

A = the faster particle.

B = the slower particle.

t = the time the particles travel until they meet.

u = the number of laps made by A in t.

v = the number of laps made by B in t.

P = the distance covered by A in t.

Q = the distance covered by B in t.

W = the length of the orbit A is on.

S = the length of the orbit B is on.

C = the speed of A = 10 m/s

D = the speed of B = 8 m/s

R = the radius of the larger orbit = 2 m

L = the radius of the smaller orbit = 1 m

Then,

$P = C \times t = 10t$, and $Q = D \times t = 8t$.

$P = W \times u$, and $Q = S \times v$.

$W = 2\pi R = 2\pi \times 2 = 4\pi$, and $S = 2\pi \times 1 = 2\pi$.

So $P = 4\pi u$, and $Q = 2\pi v$.

Thus, $4\pi u = 10t$, and $2\pi v = 8t$.

Now, once the particles have made the first rendezvous, they repeat the same until the next one.

Therefore, we take the ratio between the numbers of the laps by A and B, that is, u:v.

We have $4\pi u = 10t$, and $2\pi v = 8t$.

So $u = 10t/4\pi = 5t/2\pi$, and $v = 8t/2\pi$.

Thus, $u:v = 5:8$.

Therefore, in every time interval t in which A makes 5 laps, and B makes 8 laps, they meet at the point where the two orbits meet.

Suppose now, $u = 5$. Then, $10t = 4\pi u = 20\pi \Rightarrow t = 2\pi$ sec.

So at every $2\pi$ (approximately 6.28) seconds, the particles meet at the point where the two orbits meet.

Therefore, the two particles meet every time the faster particle makes 5 laps, and the other makes 8 laps in every $2\pi$ seconds.